Probability Models: A Tool for Modelling The Diameter of Triplochiton Scleroxylon K.Schum

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Abstract

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Diameter distributions are important tools in forest resource management and its interpretation affects silvicultural decisions. Stacy distribution is a three-parameter distribution that is transformed into two-parameter Gamma and Weibull distributions to model the diameter of Triplochiton scleroxylon K.Schum tree in Onigambari forest in the Southwestern part of Nigeria. The data exhibit positive skewness which necessitates the usage of the distributions. The parameters of the models were estimated using Maximum Likelihood Estimation method and the comparison of the models was based on goodness-of-fit statistics. The study shows that the two distributions are indistinguishable but the Gamma distribution was more accurate than the Weibull distribution based on further statistics.

***Keywords**:* Gamma Distribution, Weibull Distribution, Stacy Distribution, Triplochiton scleroxylon K.Schum, Skewness

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1 INTRODUCTION

Modeling of diameter of tree using various statistical models has been done throughout the world to give better understanding about the tree pattern and characteristics which involve the study of the diameter of the tree-level at diameter breast height measurement (Ige, 2013; Namiranian, 1990; Mataji *et al.,* 2000; Fallah *et al.,* 2000, 2006; Mohammad *et al.,* 2009; Nanang, 1998; Nord- Larson and Cao, 2006 and Mabvurira *et al.,* 2002). Forest growth modeling has been an intrinsic part of forest management planning and research for more than two centuries. The majority of models operate at the stand-level and predict stand-level variables such as basal area or dominant height to provide information needed to estimate harvesting costs, expected yield, and financial result.

Diameter Breast Height (DBH) is a standard and the most common method of measuring tree dimension apart from tree height. It can be applied to monitor the growth of trees and compare the dimensions of different trees. It is also one of the evaluation criteria for tree felling application. DBH refers to the diameter of tree trunk measured at breast level as a convenient way of measurement during which one does not need to bend his waist or climb up a ladder to take the measurement. For a more precise measurement, there is a need to standardize the "breast height". In the United States, DBH is measured at a height 4½ ft above ground. It is measured at 1.3 m above ground level. As such, in tree surveys for tree felling applications, diameters of tree trunk are measured at 1.3 m above ground in accordance with the international practice. For a tree with a clear straight or gradual tapering trunk, measuring the DBH is straight forward. However, in some other situations, there may be uncertainties as to how the DBH should be measured. Tree resources provide a large number of services such as wood, non-timber forest products, carbon Sequestration, regulation of the water cycle, soil fertility, livestock fodder, etc. Assessing these resources, whether from trees inside or outside forests, is increasingly important given the continued degradation they face and the urgent need to design and implement appropriate policies, measures and ages ascertained or expected life span for their sustainable management.

Diameter distribution and the related statistical model can play an important role in some forest science topics, including forestry and silvics, For example in some growth modelling, it is necessary to know the type of diameter distribution function and its parameters to identify the appropriate model (Mohamma *et al.,* 2009). Diameter distributions can be used to indicate whether the density of smaller trees in a stand is sufficient to replace the current population of larger trees and to help evaluate potential forest sustainability (Rubin and Manion, 2006). Diameter distributions are important tools in forest resource management. The interpretation of diameter distributions will affect silvicultural decisions, such as when to thin and how much to thin, as well as harvesting decisions, such as where and when to harvest and what kinds of equipment will be necessary. Diameter distributions are also used as inputs to growth models and sometimes are the subject of growth modeling themselves. Consequently, information about the diameter distribution for a forest stand as it is and as it may be in the future is very useful for forest management. Clutter *et al*., (1983), Borders *et al.*, (1987), Vanclay (1994), and Avery and Burkhart (2003) provide useful overviews of the uses and interpretation of diameter distributions in forest management.

Triplochiton Scleroxylon K. Schumm

Triplochiton Scleroxylon is a tropical tree of Africa. The Triplochiton Screxylon K. Schum Tree is known as “Abachi”. It is known in Nigeria as “Obeche”, in Ghana as “Wawa”, in Cameroon as “Ayous” and in Ivory Coast as “Samba”. It is used in the manufacturing of picture frames and moldings used by bespoke picture framers. Triplochiton Scleroxylon K. Schum is also a member of the Sterculiaceae and is a very important timber species in the forest areas of West Africa. There are various records on its utilization in relation to its economic value and these were collated. The total value of round and sawn wood of Triplochiton Screxylon K. Schum exported from Nigeria within this period was one hundred and twenty million, one hundred thousand Naira 120.1 m) ca.U.S.$600.5 m representing 48.7 per cent of a total of 246.6 million Naira (ca. $ 1,233 m) total earnings from the exportation of over 40 different timber species. The exportation of logs of Obeche and other species was stopped at the end of the civil war in 1970 to satisfy the increased pressure in local demand. The level of exploitation of Obeche throughout its natural range of distribution in West Africa had been high.

2.0 STUDY AREA

The study was carried out on the stands of Triplochiton scleroxylon plantation at Onigambari forest reserve South-western Nigeria. The stands examined were established in 1994. Onigambari Forest Reserve is located on latitude 70 25’ and 70 55’N and longitude 30 53’ and 30 9’E within the low land semi-deciduous forest belt of Nigeria and covers a total land area of 17,984ha. The reserve is divided into two: natural and plantation forests. The natural forest is made up of indigenous species such as *Terminalia spp, Triplochiton scleroxylon*, *Irvingia garbonensis, Treculiaafricana*, among others while the plantation forest is made up of mainly exotic species such as G*melina arborea* and *Tectona grandis*. Hence tree like Teak (Tectona grandis), Triplochition scleroxylon (Obeche), Mahogany (Khaya ivorensis), and other Agricultural crops like cocoa (Theobroma cacao), cassava (Manihot esculata) with exotic trees and crops are cultivated .

The reserve has been reduced to secondary high forest dominated by trees such as: Celtis zenkeri, Terminilia ivorensis, Triplochition scleroxylon, Terminilia superb, and Mansonia altissima and spp while the planted area is dominated by the following trees: Tectona grandis, Gmelina arborea, Mansonia altissima, Triplochiton scleroxylon. The topography of the study area is generally undulating, lying at altitude between 90mand 140m above sea level. The annual rainfall ranges between 1200mm to 1300mm spreading over March to November. The dry season is severe and the relative humidity is low and average annual temperature is about 26.4^{0}C (Larinde and Olasupo, 2011**).**

The noticeable surrounding areas area Idi Ayunre, Adebayo, Ibusogboro and Mamu. These areas are along the same equatorial belt with the study area. Onigbambari area were bounded up with the following villages; Aba-Igbagbo , gbale-asun , Ajibodu, Lagunju, Akintola, Okeseyi, Akinogbun, Amosun, Olonde ige, Olaya, Onipede. The inhabitants of the area are predominantly farmers with relatively low number of hunters. Some of the forestry practices carried out in this area includes:

(i) Planting of trees for both timber and fuel wood production.

(ii) Collection and sales of non-wood products such as leaves and bark for herbs rattance (cane), sponge, snails, leave and ropes etc.

The reserve is characterized with high humidity. Both dry and wet season are experienced in the reserve. Dry season usually lasts for 3months (December – February). The average annual rainfall is about 1140mm and average annual temperature is about 26.4^{0}C (80^{0}F). The soil type is sandy –loam and supports the growth of a wide variety of tree species herbs, shrubs as well as climbers. Drainage system structure on the plantation is very poor. The only source of water to the areas where the study was carried out is rainfall. The other source of water to these plantations is the River Ona, but the study area is a far distant from the River.

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3.0 METHODOLOGY

3.1 Unbounded Positive Continuous Distribution

The two main families of (unbounded) positive continuous distribution are the gamma-transformed family and beta-transformed family (Klugman *et al.,* 2009).

3.2 Generalized Gamma (Stacy) Distribution

Generalized gamma distributions alternatively called the gamma–Weibull distribution, it can be obtained by introducing a moment parameter in the Weibull distribution. The derivation results in a different parameterization.

Let X follow a gamma distribution $G(\alpha,1)$, the gamma transformed family is given as:

$$Y = \frac{X^{\frac{1}{\tau}}}{\lambda} \qquad \tau > 0 \qquad (1)$$

The density and distribution function in (1) are

$$f_Y(y) = \frac{\lambda^{\tau\alpha-1}}{\Gamma(\alpha)} \tau y^{\alpha\tau-1} \exp\{-(\lambda y)^{\tau}\} \qquad (2)$$

$$F_Y(y) = \frac{\Gamma(\alpha,(\lambda y)^{\tau})}{\Gamma(\alpha)} \qquad (3)$$

Where Γ ( ) denotes the incomplete gamma function (Olver *et al.*, 2010). The Gamma, Weibull and Exponential densities can be obtained from (2) by setting $\tau = 1$, $\alpha = 1$ and $\alpha = \tau = 1$

(i) The Gamma distribution

$$f_Y(y;\alpha,\lambda) = \frac{\lambda^{\alpha-1}}{\Gamma(\alpha)} y^{\alpha-1} \exp\{-(\lambda y)\}\ ,\ y>0,\ \alpha,\lambda>0 \qquad (4)$$

Where y is the diameter of the tree, α is the shape parameter, λ is the scale parameter. The Gamma cumulative distribution function is given by

$$F_Y(y) = \frac{G(\alpha,\ \lambda y)}{\Gamma(\alpha)} \qquad (5)$$

(ii) The Exponential distribution

$$f_Y(y) = \lambda \exp(-\lambda y)\ , y > 0 \qquad (6)$$

The Exponential cumulative distribution function is given by

$$F_Y(y) = 1 - \exp(-\lambda y) \qquad (7)$$

(iii) The Weibull distribution

$$F_Y(y,\tau,\lambda) = \frac{\tau}{\lambda}\left(\frac{y}{\lambda}\right)^{\tau-1} \exp\left\{-\left(\frac{y}{\lambda}\right)^{\tau}\right\}\ \ y > 0, \tau, \lambda > 0 \qquad (8)$$

The Weibull cumulative distribution function is given by

$$F_Y(y) = 1 - \exp\left\{-\left(\frac{y}{\lambda}\right)^{\tau}\right\} \qquad (9)$$

The Exponential cumulative distribution function can also be given as

$$F_Y(y) = 1 - \exp\left\{-\left(\frac{1}{\lambda}\right)\right\} \qquad (10)$$

The parameters (α, τ, λ) of the distributions can be calculated using maximum likelihood method (Seguro and Lambert, 2000; Chang ,2010).

3.3 Maximum likelihood Estimation (MLE)

The method of maximum likelihood estimator (Harter and Moore, 1965a, 1965b) is a commonly used procedure because it has very desirable properties. Let $y_1, y_2,..., y_n$ be a random sample of size n drawn from a probability density function of this random sample is the joint density of the n random variables and is a function of the unknown parameter. Thus, $L = \prod_{i=1}^{n} f(y_i;\theta)$ is the likelihood function, the maximum likelihood estimator (MLE) of θ, say θ, is the value of θ that maximizes L or equivalently, the logarithm of L. Often, but not always, the MLE of θ is a solution of $\frac{d \log\ L}{d\theta} = 0$. Where solution that are not functions of the sample values $y_1, y_2,..., y_n$ are not admissible , nor are not solution which are not in the parameter space.

3.3.1 Maximum likelihood Estimation of Gamma Distribution

Let $y_1, y_2,..., y_n$ be a random sample from the gamma distribution. The likelihood function is

$$L(y;\alpha,\lambda) = \prod_{i=1}^{n} f_Y(y;\alpha,\lambda) \qquad (11)$$

$$= \frac{\lambda^{n\alpha}}{[\Gamma(\alpha)]^n} \exp\left(-\lambda \sum_{i=1}^{n} y_i\right)\left(\prod_{i=1}^{n} y_i\right)^{\alpha-1}$$

The log-likelihood function is

$$\log L(\alpha,\lambda) = \log\left[\frac{\lambda^{n\alpha}}{[\Gamma(\alpha)]^n} \exp\left(-\lambda \sum_{i=1}^{n} y_i\right)\left(\prod_{i=1}^{n} y_i\right)^{\alpha-1}\right] \qquad (12)$$

$$= \sum_{i=1}^{n} \left[-\log \Gamma(\alpha) + \alpha \log \lambda + (\alpha - 1) \log y_i - \lambda y_i \right]$$

$$= -n \log \Gamma(\alpha) + n\alpha \log \lambda + (\alpha - 1) \sum_{i=1}^{n} \log y_i - \lambda \sum_{i=1}^{n} y_i$$

Simplifying the partial derivative of the log-likelihood function with respect to α and λ gives

$$\frac{\partial \log L(y \mid \alpha, \lambda)}{\partial \alpha} = \frac{n\alpha}{\lambda} - \sum_{i=1}^{n} y_i \qquad (13)$$

$$\frac{\partial \log L(y \mid \alpha, \lambda)}{\partial \alpha} = n \log \lambda - \frac{n}{\Gamma(\alpha)} \frac{\partial \Gamma(\alpha)}{\partial \alpha} + \sum_{i=1}^{n} \log y_i \qquad (14)$$

Thus

$$n \log \lambda - \frac{n}{\Gamma(\alpha)} \frac{\partial \Gamma(\alpha)}{\partial \alpha} + \sum_{i=1}^{n} \log y_i = -n\psi(\alpha) + n \log \lambda + \sum_{i=1}^{n} \log y_i \qquad (15)$$

where

$$\psi(\alpha) = \frac{\partial \log \Gamma(\alpha)}{\partial \alpha}$$

Setting the derivative to zero and solve for α and λ, the MLE are

$$\hat{\lambda} = \frac{\alpha}{\bar{y}} \qquad (16)$$

$$n \log\left(\frac{\alpha}{\bar{y}}\right) - \sum_{i=1}^{n} \log y_i = \frac{n}{\Gamma(\alpha)} \frac{\partial \Gamma(\alpha)}{\partial \alpha}\bigg|_{\alpha = \bar{\alpha}} \qquad (17)$$

A numerical procedure is needed for its computation

The log-likelihood has second derivatives called the fisher information

$$\frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \lambda \partial \lambda} = \frac{-n\alpha}{\lambda^2} \qquad (18)$$

$$\frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \lambda \partial \alpha} = \frac{-n}{\lambda} = \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \alpha \partial \lambda} \qquad (19)$$

$$\frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \alpha \partial \alpha} = -n\psi'(\alpha) \qquad (20)$$

The Fisher information and the inverse of the Fisher information is

$$I(\theta)^{-1} = \left\{ -E\left[\frac{\partial^2 \log L}{\partial \theta_i \partial \theta j} \right] \right\}^{-1} \qquad (21)$$

Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \lambda \partial \lambda} & \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \lambda \partial \alpha} \\ \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \alpha \partial \lambda} & \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \alpha \partial \alpha} \end{bmatrix} \qquad (22)$$

$$I(\hat{\theta})^{-1} = V = \begin{bmatrix} \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \lambda \partial \lambda} & \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \lambda \partial \alpha} \\ \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \alpha \partial \lambda} & \frac{\partial^2 \log L(y \mid \alpha, \lambda)}{\partial \alpha \partial \alpha} \end{bmatrix}^{-1} \qquad (23)$$

$$V = \begin{bmatrix} V_{11} & 0 \\ 0 & V_{22} \end{bmatrix} \qquad (24)$$

Interval Estimation

The asymptotic distribution of the maximum likelihood estimator of the parameters is

$$I(\hat{\theta})^{-1} = V = \begin{bmatrix} V_{11} & 0 \\ 0 & V_{22} \end{bmatrix} \qquad (25)$$

The asymptotic distribution of the maximum can be written as follows (Miller *et al.*, 1981)

$$\left[(\hat{\alpha} - \alpha), (\hat{\lambda} - \lambda) \right] \sim N_2(0, V) \qquad (26)$$

V involves the parameters α and λ, we replace the parameters by the corresponding MLEs in order to obtain an estimate of V, which is denoted by $\hat{V}$.

The appropriate $100(1-\alpha)\%$ confidence intervals for α and λ are determined respectively as

$$\left[\hat{\alpha} \pm Z_{\alpha/2} \sqrt{\hat{V}_{11}}, \quad \hat{\lambda} \pm Z_{\alpha/2} \sqrt{\hat{V}_{11}} \right] \qquad (27)$$

3.3.2 Maximum likelihood Estimation of Weibull Distribution

Let $y_1, y_2, ..., y_n$ be a random sample from the Weibull distribution. The likelihood function is

$$L(y; \tau, \lambda) = \prod_{i=1}^{n} f_Y(y; \tau, \lambda) \qquad (28)$$

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$$=\prod_{i=1}^{n}\left[\frac{\tau}{\lambda}\left(\frac{y}{\lambda}\right)^{\tau-1}\exp\left\{-\left(\frac{y}{\lambda}\right)^{\tau}\right\}\right] \qquad (29)$$

$$=\frac{\tau^{\alpha}}{\lambda^{n\tau}}\left[\prod_{i=1}^{n} y_i\right]^{\tau-1}\exp\left(-\sum_{i=1}^{n}\frac{y_i}{\lambda}\right)^{\tau} \qquad \lambda,\tau>0$$

$$\log L(\tau,\lambda)=\log\left[\frac{\tau^{\alpha}}{\lambda^{n\tau}}\left[\prod_{i=1}^{n} y_i\right]^{\tau-1}\exp\left(-\sum_{i=1}^{n}\frac{y_i}{\lambda}\right)^{\tau}\right] \qquad (30)$$

$$=n\log\tau-n\log\lambda+(\lambda-1)\sum_{i=1}^{n}\frac{y_i}{\lambda}-\sum_{i=1}^{n}\left(\frac{y_i}{\lambda}\right)^{\tau} \qquad (31)$$

Simplifying the partial derivative of the log-likelihood function with respect to τ and λ gives

$$\frac{\partial\log L(y\,|\,\tau,\lambda)}{\partial\lambda}=-\frac{n\tau}{\lambda}+\frac{\tau}{\lambda}\sum_{i=1}^{n}\left(\frac{y_i}{\lambda}\right)^{\tau} \qquad (32)$$

$$\frac{\partial\log L(y\,|\,\alpha,\lambda)}{\partial\tau}=\frac{n}{\tau}+\sum_{i=1}^{n}\log\left(\frac{y_i}{\lambda}\right)^{\tau}\log\left(\frac{y_i}{\lambda}\right) \qquad (33)$$

The log-likelihood has second derivative called the Fisher Information which is

$$\frac{\partial^2\log L(y\,|\,\alpha,\lambda)}{\partial\lambda\partial\lambda}=\frac{\tau(\tau+1)}{\lambda^2}\sum\left(\frac{y_i}{\lambda}\right)^{\tau}-\frac{n\tau}{\lambda^2} \qquad (34)$$

$$\frac{\partial^2\log L(y\,|\,\tau,\lambda)}{\partial\lambda\partial\tau}=\frac{\sum_{i=1}^{n}\left[1+\left(\frac{y_i}{\lambda}\right)^{\tau}\left\{1+\tau\log\left(\frac{y_i}{\lambda}\right)\right\}\right]}{\lambda} \qquad (35)$$

$$\frac{\partial^2\log L(y\,|\,\tau,\lambda)}{\partial\lambda\partial\tau}=\frac{\partial\log L(y\,|\,\alpha,\lambda)}{\partial\tau\partial\lambda}$$

$$\frac{\partial^2\log L(y\,|\,\tau,\lambda)}{\partial\tau\partial\tau}=\frac{n}{\tau^2}+\sum_{i=1}^{n}\left(\frac{y_i}{\lambda}\right)^{\tau}\left\{\log\left(\frac{y_i}{\lambda}\right)\right\}^2 \qquad (36)$$

The Fisher information matrix is useful for the purpose of calculating the interval estimates, asymptotic variances and covariances, and test of hypothesis. The inverse of the Fisher information is

$$I(\theta)^{-1}=\left\{-E\left[\frac{\partial^2\log L}{\partial\theta_i\partial\theta j}\right]\right\}^{-1} \qquad (37)$$

Interval Estimation

The asymptotic distribution of the maximum likelihood estimator of the parameters is

$$I(\hat{\theta})^{-1}=V=\begin{bmatrix}V_{11} & 0\\ 0 & V_{22}\end{bmatrix} \qquad (38)$$

The asymptotic distribution of the maximum can be written as follows

$$\left[(\hat{\tau}-\tau),\,(\hat{\lambda}-\lambda)\right]\ \sim N_2(0,V) \qquad (39)$$

V involves the parameters α and , we replace the parameters by the corresponding MLEs in order to obtain an estimate of V, which is denoted by $\hat{V}$.

The appropriate $100(1-\alpha)\%$ confidence intervals for α and λ are determined respectively as

$$\left[\hat{\tau}\pm Z_{\alpha/2}\sqrt{\hat{V}_{11}},\quad \hat{\lambda}\pm Z_{\alpha/2}\sqrt{\hat{V}_{11}}\right] \qquad (40)$$

3.4 Judgment Criterion

In order to check how accurate a theoretical probability density function fits with observational data, the judgment criterion error is shown below:

Kolmogorov-Smirnov test given as

$$Q=\max\left|T(y)-O(y)\right| \qquad (41)$$

Where $T(y)$ is the cumulative distribution function for diameter not exceeding y, calculated from a theoretical function; similarly $O(y)$ is calculated from observation data

4.0 RESULTS AND DISCUSSIONS

The diameter distribution of Triplochiton Scleroxylon K.schum in Onigambari forest in the Southwestern part of Nigeria was examined as 302 Triplochiton Scleroxylon K.Schum were sampled and modeled using Stacy distribution which is transformed into two-parameter Gamma and Weibull distributions. The mean, standard deviation, skewness and kurtosis for the dataset were shown in (Table 1). The mean is 58.83 with a standard deviation of 33.50. The distribution is skewed to the right (+0.93), this is evident in the histogram of the dataset. The parameters of the distributions were estimated using maximum likelihood method, the results were shown in (Table 2) while the results of AIC, BIC, log-likelihood, and K-S statistics were shown in (Tables 2 & 3) which measure maximum deviation between observed and predicted CDFs. The results indicate that both distributions are appropriate and accurate to model and predict the diameter of Triplochiton Scleroxylon K.schum based on the estimated values. This is also evident in the histogram with the fitted densities (Figures 1, 2 & 9). The Q-Q plots and P-P plots were obtained and shown in Figures 4, 5, 6 & 7. The more points that are close to the line, the better fit the distribution. The plots revealed that both distributions sufficiently fit the dataset without overfitting. Comparison of the fit of the two distributions is possible as both pdfs have the same number of parameters. The closeness in the fits is most noticeable in the plots and unsurprisingly the estimated values of AIC, BIC and log-likelihood are almost identical. Assessment of the quality of fit based on K-S and further statistics indicates that Gamma distribution seems to be a reasonable approximation of the data.

Table 1: Descriptive Statistics from the Data

Sample	Mean	Standard Dev.	Skewness	Kurtosis
302	58.8370	33.5028	0.9287	0.7714

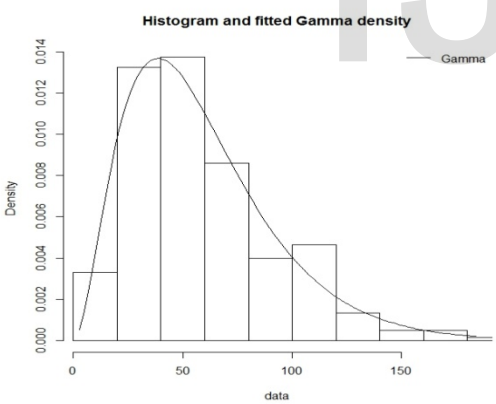


Fig. 1 The Plot of Gamma distribution superimposed with parameters estimated as $(\hat{\alpha}\ ,\hat{\lambda}) = (2.9112\,, 0.0495)$

Table 2: Parameter of the Models

Gamma			**Weibull**		
Parameter	Estimate	Std.Error	**Parameter**	Estimate	Std.Error
Shape (α)	2.9112	0.2243	**Shape** (τ)	1.8478	0.08124
Scale (λ)	0.0495	0.0042	**Scale** (λ)	66.3828	2.1819
Log-likelihood	-1460.064		**Log-likelihood**	-1460.91	
AIC	2924.129		**AIC**	2925.819	
BIC	2931.55		**BIC**	2933.24	

Table 3: Results of the tests for the goodness of fit

	Gamma	Weibull
Kolmogorov-Smirnov	0.0288	0.0422
p-value	0.9632	0.6541

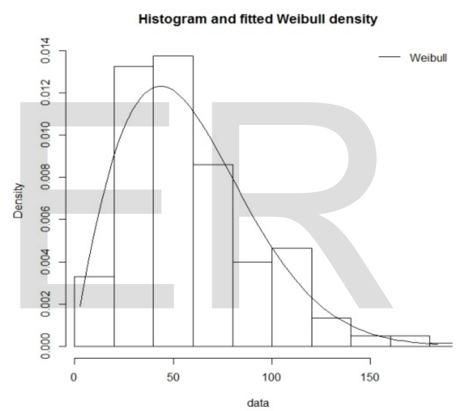
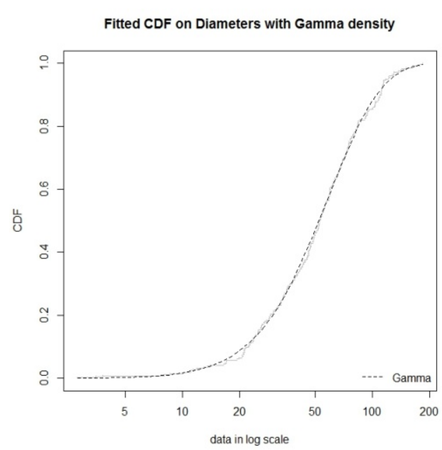


Fig. 2 The Plot of Weibull distribution superimposed with parameters estimated as $(\hat{\tau}\ ,\hat{\lambda}) = (1.8478,\quad 66.3828)$.



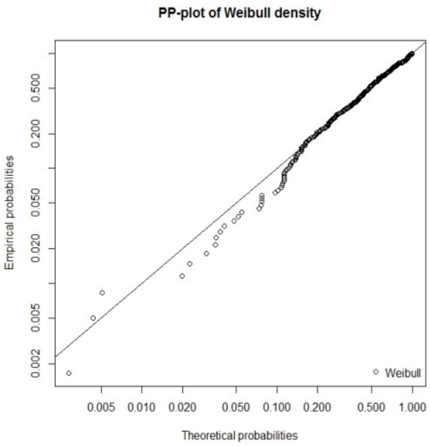


Fig. 3 Cummulative Distribution Graph for Gamma Density

Fig. 6 Probability Plot of Weibull Distribution

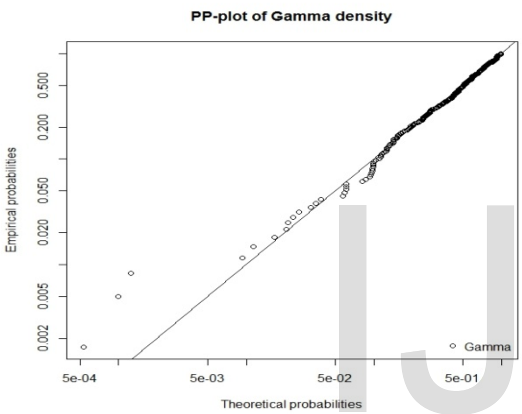


Fig. 4 Probability Plot of Gamma Distribution

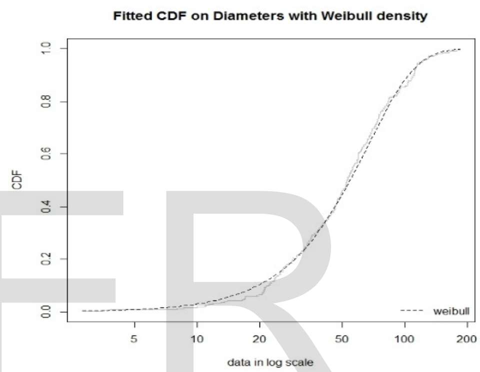


Fig. 7 Cummulative Distribution Graph for Weibull Density

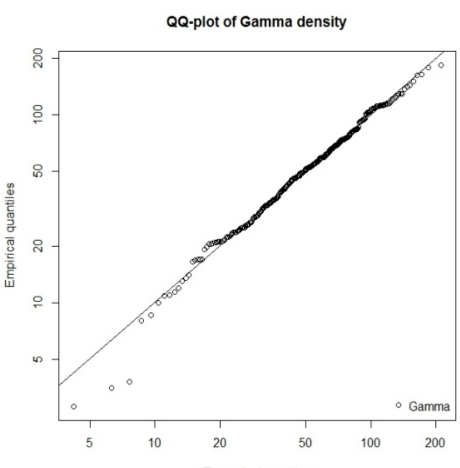


Fig. 5 Quantile-Quantile Plot of Gamma Distribution

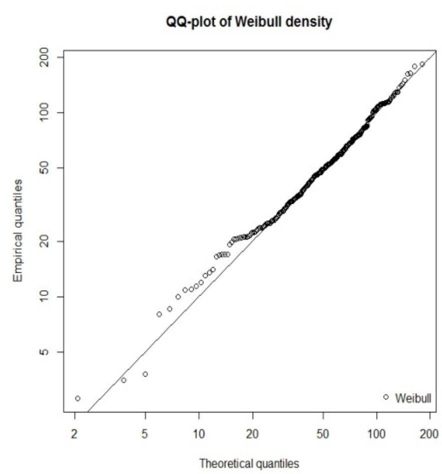


Fig. 8 Quantile-Quantile Plot of Weibull Distribution

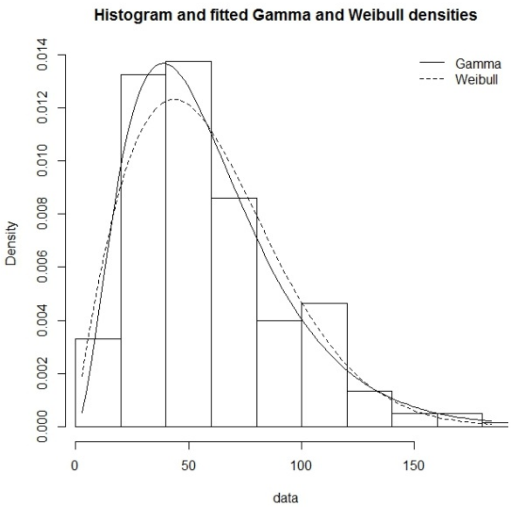


Fig. 9 Comparison of two-parameter Gamma and Weibull distributions fitted curves to the diameter dataset.

5.0 CONCLUSIONS

The data exhibit positive skewness which necessitates the usage of the distributions, two distribution recovered from Stacy distribution (Gamma and Weibull) were used to model the diameter of Triplochiton Scleroxylon K.schum tree in Onigambari forest in the Southwestern part of Nigeria. The two models are indistinguishable but Gamma distribution performs better than Weibull distribution.

REFERENCES

[1] Chang, T.P.(2010):*Performance comparison of six numerical methods in estimating Weibull parameters for wind energy application*. Applied Energy in press, doi: 10.1016/j.apenergy.2010.06.018.

[2] Fallah, A., Zobeiry, M., Jaziriee, M.H. and Marvie Mohadjer, M.R. (2000): Iranian Journal natural resources,53, No.3: 251-260. (in Persian).

[3] Harter, H. L. and Moore (1965b): "*Maximum likelihood estimation of the parameters of Gamma and Weibull populations from complete and from censored samples*" Technometrics, Vol. 7, No. 4.

[4] Harter, H. L. and Moore (1965a): "*Point and interval estimator based on order statistics for the scale parameter of a Weibull population with known shape parameter"* Technometrics, Vol. 7, No. 3.

[5] Ige, P.O; Akinyemi, G.O and Abi, E.A. (2013): *Diameter Distribution Models for Tropical Natural Forest trees in Onigambari Forest Reserve.* ,Journal of Natural Sciences Research ISSN 2224-3186 (Paper) Vol.3, No.12, 2013.

[6] Klugman, S.A., Panjer, H.H. and Willmot, G.E. (2009): *Loss Models: From Data to Decisions*, Wiley Series in Probability and Statistics, New York.

[7] Klugman, S.A., & Parsa, R. (1999): *Fitting bivariate loss distribution with copulas, Insurance: Mathematics and Economics 24, 139-148 Loss Models: From Data to Decisions*, Wiley Series in Probability and Statistics, New York.

[8] Mabvurira, D. M. Maltamo and .Kangas, A. (2002). *Predicting and calibrating diameter distributions of Eucalyptus grandis (Hill) Maiden plantations in Zimbabwe.* New Forests 23:207-223.

[9] Mattaji, A. Hojjati, S.M., and Namiranian, M. (2000). Iranian Journal natural resources 53, No.2: 165-172. (in Persian).

[10] Mohammad Alizadeh, Kh , Zobeiry, M. ,Namiranian, M. Hoorfar, A. and Marvie Mohadjer, M..R. (2009) *Fitness of distribution of diameter of breast height by using some atastistic distribution models.* Iranian Journal of Forest and Poplar Research, , 17, No.1, 116-124.

[11] Namiranian, M. (1990): *The application of probability theories in determining tress distribution in different diameters levels.* Iranian Journal natural resources, 44: 93-108.

[12] Nanang, D.M., (1998). *Suitability of The Normal, Log - normal and Weibull distribution for fitting diameter distribution of neem Plantations in Northern Ghana, Forest Ecology and Management,* 103: 1-7.

[13] Nord-Larson and Cao, Q.V., (2004). *Predicting Parameter of Weibull function for modeling diameter distribution, Forest science, 50(5):682-985.*

[14] Rubin,B.D; Manion, P.D, (2006). *Forest Ecology and Management, 222: 427-438.*

[15] Robert L. Bailey and . Dell,T. R (1973 / 99) . Forest Sci. 19:97-104.volume 19, number 2,

[16] Stacy, E.W. (1962). *A generalization of the gamma distribution. Annals of Mathematical Statistics, 33, 1187–1192.*

[17] Seguro, J.V. and Lambert , T.W. (2000): *Modern estimation of the parameters of the Weibull wind speed distribution for wind energy analysis.* Journal of Wind Engineering and Industrial Aerodynamics, 85, 75-84.